

TEMPERATURE COMPENSATED SAPPHIRE RESONATOR FOR ULTRA-STABLE OSCILLATOR CAPABILITY AT TEMPERATURES ABOVE 77 KELVIN*

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Abstract

We report on the design and test of a whispering gallery sapphire resonator for which the dominant (WGH_{n11}) microwave mode family shows frequency-stable, compensated operation for temperatures above 77 kelvin. The resonator makes possible a new ultra-stable oscillator (USO) capability that promises performance improvements over the best available crystal quartz oscillators in a compact cryogenic package. A mechanical compensation mechanism, enabled by the difference between copper and sapphire expansion coefficients, tunes the resonator to cancel the temperature variation of sapphire's dielectric constant. In experimental tests, the WGH_{811} mode showed a frequency turn-over temperature of 87 K in agreement with finite element calculations. Preliminary tests of oscillator operation show an Allan Deviation of frequency variation of $1.4 - 6 \times 10^{-12}$ for measuring times $1 \text{ second} \leq \tau \leq 100 \text{ seconds}$ with unstabilized resonator housing temperature and a mode Q of 2×10^6 . We project a frequency stability 10^{-14} for this resonator with stabilized housing temperature and with a mode Q of 10^7 .

1 Introduction

A sapphire whispering gallery resonator consists of a wheel or disk of sapphire inside a cylindrical metallic shielding can. By confining resonating rf fields to the sapphire element, these resonators effectively eliminate metallic conduction losses – and so make possible resonators which are only limited by performance of the sapphire itself. The sapphire is typically oriented with its crystal c -axis along the central axis of the container in order to achieve cylindrical symmetry for the excited resonance modes.

Whispering gallery electromagnetic modes can be divided into families depending on their field configuration, and further characterized by the number of full waves n around the perimeter of the sapphire element. The modes are doubly degenerate, with azimuthal phase of the two sub-modes differing by 90

degrees. Modes typically used are the WGH_{n11} family for wheel resonators and the WGE_{n11} family for flat disks, where $n \geq 5$.

With very high microwave quality factors (Q's) at easily reached cryogenic temperatures, the sapphire resonators already make possible excellent phase noise performance. In principle, the high Q values also make possible high frequency stability – if the resonator itself were stable[1–6]. However, temperature fluctuations in the sapphire cause unwanted frequency fluctuations. If these frequency variations could be cancelled or *compensated*, high stability could be achieved.

The resonator Q's increase rapidly as the temperature is cooled, from approximately $Q = 300,000$ at room temperature to 30 million at 77 Kelvin (for X-band frequencies $\approx 8 \text{ GHz}$). This compares to Q values of 1 to 2 million for the best available crystal quartz oscillators, and 10,000 to 20,000 for metallic microwave cavities. When coupled with low noise microwave circuitry, the high sapphire Q could make

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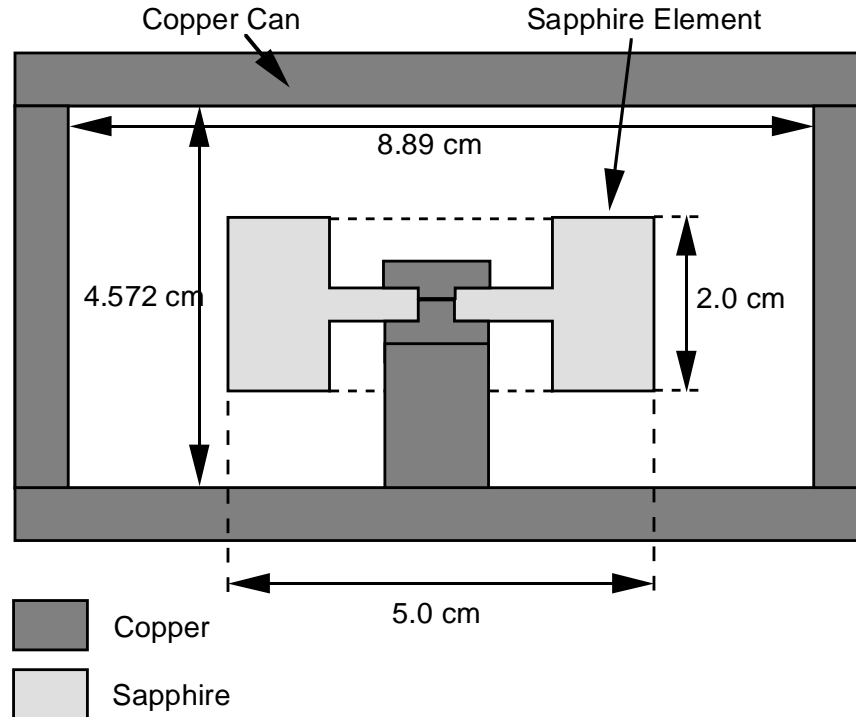


Figure 1: Whispering gallery sapphire resonator (cylindrical cross section)

possible a frequency stability as low as 10^{-14} [1]. Such a stability would be 20 times better than that achievable by quartz oscillators of the highest quality, which presently give a stability of 2×10^{-13} .

Finite element calculations of frequencies and frequency tuning rates with temperature were made using the CYRES 2-D program [see companion paper, this conference]. A comparison of calculated and measured tuning rates at 77K shows excellent agreement. In particular, both calculation and experiment show a weakening of the compensation effect for higher mode numbers that was not predicted by a simple circuit model that was initially used to design the resonator.

2 Background

Various approaches have been developed to reduce the thermal variation in electromagnetic or acoustic resonators and so achieve high frequency stability. Compensated operation for bulk acoustic wave (BAW) quartz oscillators is achieved by means of an appropriate choice of orientation for the quartz crys-

tal. This is possible due to a very strong variability of acoustic parameters with crystal direction. The *electromagnetic* sapphire resonators have a much smaller anisotropy ($\approx 35\%$) and no sign reversal for any of its thermal dependencies. In fact, up to the present time useful compensation of sapphire resonators has only been possible at liquid helium temperatures, where incidental or added paramagnetic impurities give an effective compensating effect[2,7]. But helium temperature operation is expensive, and impractical for most applications. A compensation mechanism at 77 kelvin or above would allow liquid nitrogen could be used as the coolant in a very much smaller and less expensive system.

Temperature sensitivity of the operating frequency is characteristic of all electromagnetic and acoustic (piezoelectric) resonators due to thermal variation of the size, dielectric constants, speed of sound, etc. for solid state materials. Variation of these parameters is typically parts in 10^4 to 10^5 per Kelvin. Thus, achieving resonator stabilities of 10^{-13} to 10^{-14} would require *nano-degree* temperature stability – an impossible task.

Available techniques for higher stability and re-

duced thermal variation in resonator frequencies are:

- Very low cryogenic temperatures ($T < 10$ Kelvin) can be used to “freeze out” the thermal-induced variation, which varies as T^3 as the components are cooled. This technique has been successfully applied to superconducting[9], superconductor-on-sapphire[7], and sapphire whispering gallery resonators[2]. However, the very low temperature required makes such systems large and expensive, and therefore impractical for most applications.
- An inherently weak tuning mechanism may be used at the lowest temperatures to provide complete cancellation. In this way paramagnetic impurities can compensate the thermal variation in sapphire resonators for $T \leq 6$ Kelvin[1,2], but again, operation at such temperatures is impractical for most applications.
- The differing thermal coefficients for various properties of the resonator material can be played against each other in such a way that, for some operating temperature, thermal frequency variations are compensated or cancelled. Piezoelectric quartz resonators are compensated in this way by an appropriate orientation of this strongly anisotropic crystal (e.g. “SC” or “AT” cut quartz resonators) [10]. Unfortunately, an orientation-dependent cancellation does not occur for electromagnetic resonators where the anisotropy is much smaller (the temperature dependencies vary by only $\approx 30\%$ as the orientation is changed).
- A resonator may be constructed using several similar materials with compensating thermal characteristics. For example, dielectric resonators for DRO’s are typically stabilized by use of several materials with thermal dielectric variations of opposite sign [11].
- A mechanical tuning mechanism may be driven by thermal expansion coefficients of the construction materials. This mechanism has been previously applied to a sapphire resonator at room temperature using a highly reentrant geometry to achieve very low phase noise and a stability of 4×10^{-8} at $\tau = 10$ seconds [3]. Ultra-high stability was probably precluded by susceptibility of the design to thermal gradients. This is also the methodology of the present work.

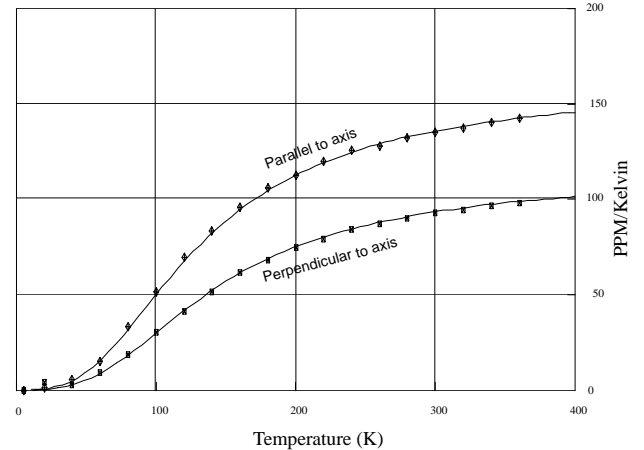


Figure 2: Temperature coefficient of the dielectric constant of sapphire for the components parallel and perpendicular to the c-axis.

3 Material Properties

Sensitivity of the sapphire resonator’s frequency to temperature is due to several factors.

- Variation of the dielectric constants with temperature is the largest factor. As shown in Fig. 2, they vary by 80–140 parts per million (PPM) per Kelvin at room temperature (300 Kelvins)[12]. The resulting frequency change is just half this value, or 40–70 PPM/Kelvin (since $f \propto 1/\sqrt{\epsilon}$).
- The expansion coefficients of sapphire impact the frequency directly. As shown in Fig. 3 they give rise to a frequency change of 5–6 PPM/Kelvin.
- Thermal expansion of the copper containing can is a small but significant factor. Because microwave energy density at the walls is greatly reduced, (typically 100 to 10,000 times, to enable a high sapphire Q) the frequency sensitivity to can size is reduced by this same factor. Thus the 15 PPM/Kelvin copper expansion (Fig. 3) is reduced to 0.15 PPM/Kelvin or smaller [13].

Short term thermal stability of approximately 1 μ Kelvin can be attained at room temperature or at 77K [14]. However, even this very low variability, when coupled with a sapphire frequency sensitivity of $\approx 6 \times 10^{-5}/\text{K}$ (room temperature) or $\approx 1.25 \times 10^{-5}/\text{K}$ (77K)[13], gives frequency variations of 1 –

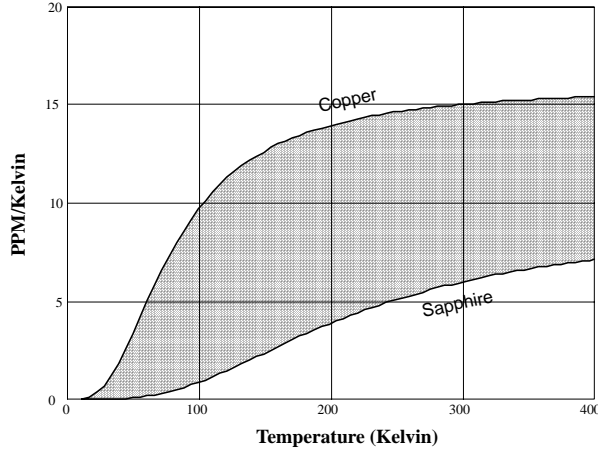


Figure 3: Thermal expansion coefficients for copper and sapphire. Differential motion (shaded area) can be used to tune a variable frequency resonator.

6×10^{-11} . These values are substantially worse than attainable with excellent quality quartz oscillators.

On the other hand, if this sensitivity could be compensated to first order, the remaining response is a second-order effect of $\approx 10^{-7}/K^2$. If the temperature could be kept within only .01K of the compensation point, the remaining linear sensitivity would be $\leq 1 \times 10^{-9}/K$. Coupling this sensitivity value with the achievable temperature stability of $1 \mu\text{Kelvin}$ would result in a frequency stability of $\leq 10^{-15}$, a very attractive prospect.

Because there are no available internal compensation mechanisms that would give an effectively unchanging dielectric constant in the sapphire itself, we must look for solutions where one physical part of the resonator structure compensates for the effect of a different part.

In particular, if the thermal expansion profile of a second construction material were sufficiently different from sapphire, and if a mechanism could be found to use this difference to give a frequency variation of opposite sign to that of the sapphire itself, a practical compensated resonator could be constructed.

A consequence of this type of design is that the various parts must be in excellent thermal contact with each other. For example, a temperature *differential* of $1\mu\text{K}$, between the parts would give rise to the same (large) $1 - 6 \times 10^{-11}$ variation alluded to above.

Fortunately, sapphire has one of the highest thermal conductivities for any solid material in the 77K – 300K temperature range, together with relatively low thermal mass. Thus, sapphire could be mated with some other high-conductivity material in a composite resonator with a very short thermal time constant and overall high conductivity to provide a structure with high immunity to internal temperature gradients.

Since sapphire's expansion coefficient is relatively small compared to most materials, a natural choice is for a second material with a *larger* expansion coefficient. Figure 3 shows a comparison between sapphire and copper – a likely candidate by virtue of its high thermal conductivity. The **difference** between sapphire and copper values, as shown in the figure is the driving force for our compensation mechanism. It is useful to compare this difference with the temperature coefficients of the dielectric constant as shown in Figure 2. Such a comparison shows that the compensation task is much easier at 77K than 300K, since, dielectric coefficient variations are strongly reduced as the temperature decreases from 300K to 77K, while the copper-sapphire expansion difference holds constant.

A comparison of the *magnitudes* of the two effects shows that a very effective tuning mechanism is required to achieve compensation. However, at 77K the task does not seem out of reach. Here the difference between sapphire and copper expansion coefficients is

$$\frac{1}{x} \frac{\partial x}{\partial T} = \sigma_c(77K) - \sigma_s(77K) = 7 \text{ PPM/Kelvin},$$

while the dielectric tuning effect (one half of the dielectric constant variation, averaging perpendicular and parallel components) is

$$\frac{1}{\omega} \frac{\partial \omega}{\partial T} = 13.5 \text{ PPM/Kelvin}.$$

Combining these two equations we find that the required tuning sensitivity is given by

$$\left| \frac{\delta \omega}{\omega} \right| = \frac{13.5}{7} \times \left| \frac{\delta x}{x} \right|. \quad (1)$$

That is, differential thermal expansion between copper and sapphire could be used to compensate the dielectric constant variation in sapphire at 77 Kelvin if a mechanism could be found that is able to tune about twice (actually 13.5/7) as much as it moves on a fractional basis, comparing Hertz per Hertz with centimeters per centimeter.

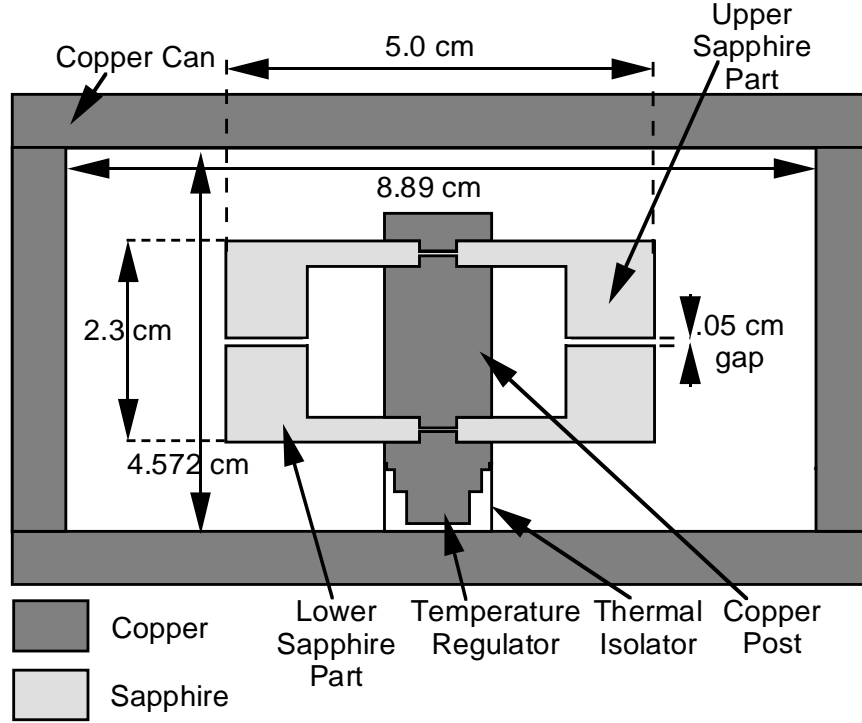


Figure 4: Compensated sapphire resonator.

It is worth noting that compensation at room temperature is much more difficult. It could be accomplished by the use of a material with a greater coefficient of expansion, such as zinc, or by the use of relatively extremely geometries. A comparison of Figs. 2 and 3 shows that increasing the temperature from 77 to 300 Kelvins increases the required tuning sensitivity by more than 4 times.

4 A Compensated Resonator

Figure 4 shows a composite microwave resonator that uses thermal expansion in an added non-sapphire tuning element to compensate for sapphire's thermal frequency variation. The resonator consists of two sapphire parts (each approximately half the thickness of a conventional whispering gallery resonator) separated by a copper post which has an expansion coefficient larger than that of the sapphire. If the gap between the two parts is small the resonant frequencies of some of the whispering gallery modes are strongly tuned as the gap spacing changes. However,

for available materials, a weak tuning effect results if the post is only as tall as the gap spacing. Thus the sapphire parts must be made re-entrant, so that the post is approximately as tall as the entire resonator itself. In this case a strong thermal tuning effect, due to the *difference* between post-material and sapphire expansion coefficients can completely cancel sapphire's inherent frequency variation. The post can be made of copper for compensation at temperatures up to about 100 Kelvin, while materials with higher expansion coefficients (e.g. zinc) could be used up to room temperature.

In order to achieve high stability in an oscillator, the high Q of the sapphire resonator must not be degraded by the presence of the post. Furthermore, because the compensating tuning effects are due to physically distinct parts, thermal gradients must be minimized, requiring high thermal conductivity through the post. We find that a copper post of approximately 20–30% of the sapphire diameter provides the required thermal conductivity. An axial position for the post minimizes any Q degradation, since electromagnetic energy in the modes is concentrated near the outer perimeter of the sapphire disks.

The compensation mechanism can be understood

as follows. As the temperature is e.g. raised, the mode frequencies are *lowered* due to the increasing dielectric constant and thermal expansion of the sapphire (Figure 2). However, the gap is widened due to the large thermal expansion of the copper post (Figure 3). The resulting increase in vacuum gap volume (dielectric constant $\epsilon = 1$ compared to ≈ 10 for sapphire) tends to *raise* the frequencies. These cancelling effects can give rise to complete compensation at some temperature.

As previously discussed, the sapphire element itself is the primary temperature-dependent element in this resonator. Thus, the compensated central sub-assembly (consisting of sapphire parts and the copper post) is thermally isolated from the copper can, and held at a stabilized operating temperature above 77 K by action of a small heater and thermometer (not shown). The exact temperature depends on the experimentally determined turnover temperature for the sapphire resonator subassembly. The can's temperature must also be controlled for the highest frequency stability, even though its thermal sensitivity is 100 to 10,000 times reduced from that of the sapphire. This is accomplished by means of a second heater/thermometer feedback system (not shown) that stabilizes the can temperature to a value just slightly above 77 K.

4.1 Variation of mode Frequency with Gap Spacing

As indicated in Eq. 1, the copper/sapphire composite resonator requires a high tuning sensitivity in order to achieve compensation at 77 Kelvins. We present two different approaches to evaluate the sensitivity of the frequency of the fundamental WGH_{n11} mode to changes in a small gap at the resonator center plane.

- With a knowledge of the mode configuration, we can estimate this sensitivity using simple *circuit models* that incorporate the resonator dimensions in a natural way. This approach has the advantage of illuminating the qualitative features of the design problem.
- A *finite element* computational technique can be used to estimate mode frequencies, both with and without a gap. Accuracy of this methodology depends on the number of *nodes* used to characterize the geometry, with fields being evaluated only at the node points, and with the

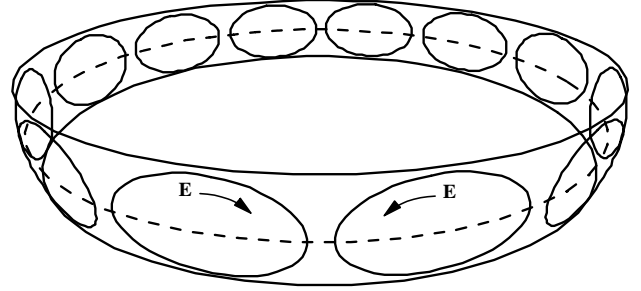


Figure 5: Electric field configuration for the WGH_{611} mode in a sapphire ring resonator. A horizontal gap at the dashed center line will cut the loops of electric field and strongly tune the mode frequency. Other modes, e.g. the WGE_{611} mode with horizontal loops, would not be strongly affected by such a gap.

space between fitted to a simple power law behavior.

We know that the magnitude of the tuning can be relatively large, as required by Eq. 1, because electromagnetic boundary conditions can give rise to larger energy in the gap region than in the (high ϵ) sapphire. In particular, this is true for modes with large electric fields perpendicular to the gap such as the WGH_{n11} mode family chosen for our application.

4.1.1 Gap Sensitivity Estimation by Circuit Analysis

In order to demonstrate that our approach is sound, the circuit analysis approach is used to estimate a lower bound for the tuning sensitivity. The WGH_{n11} mode is characterized by a chain of approximately elliptical loops of electric field in the $z - \phi$ plane (as shown in Fig. 5) linked by loops of magnetic field in the $r - \phi$ plane. Because of the continuity of the electric field lines and of displacement current, we can estimate the effect of the gap using a simple series-capacitance model. Conventional circuit analysis gives $\omega = \sqrt{1/LC}$, and assuming that any change in effective inductance is small we find:

$$\frac{\Delta\omega}{\omega} \approx -\frac{1}{2} \frac{\Delta C}{C}.$$

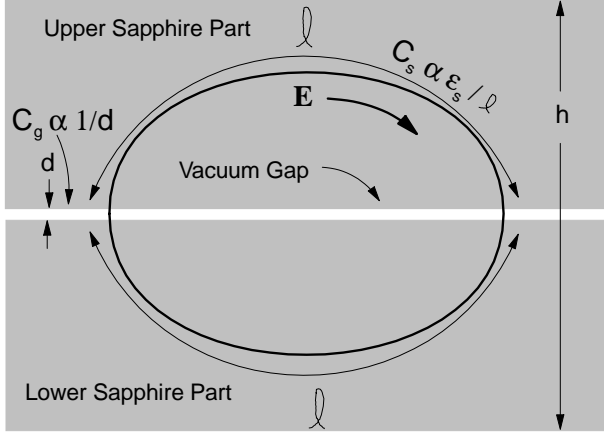


Figure 6: Detail view of one E-field loop showing elements of series-capacitance model.

Each electric field loop traverses a path ℓ through the sapphire with dielectric constant ϵ_s and then a gap distance d with $\epsilon = 1$ as shown in Fig 6. Because of the continuity of displacement current[15] we can approximate the sapphire capacitance as $C_s \propto \epsilon_s/\ell$ and the gap capacitance as $C_g \propto 1/d$. By combining these capacitances in series we estimate the dependence of the capacitance C on the gap spacing d as:

$$\frac{\Delta C}{C} \approx -\frac{d\epsilon_s}{\ell + d\epsilon_s}.$$

We estimate a lower bound for the tuning effect by assuming that the loops are as long as possible, touching both the resonator top and bottom. Approximating the elliptical loops as circular the loop length becomes

$$\ell \approx \frac{\pi}{2} \times h$$

where h is the sapphire height. Combining these two equations in the limit of small g , shows the frequency sensitivity in terms of distance sensitivity to be

$$\frac{\Delta\omega}{\omega} \approx -\frac{1}{2} \frac{\Delta C}{C} \approx \frac{1}{2} \frac{g\epsilon_s}{\ell} \approx \frac{\epsilon_s}{\pi} \times \frac{g}{h}$$

or

$$\left| \frac{\delta\omega}{\omega} \right| \geq \frac{\epsilon_s}{\pi} \times \left| \frac{\delta x}{x} \right| \quad (2)$$

where $\delta x/x \equiv \delta g/h$ is the fractional variation of the resonator height.

Since $\epsilon_s \approx 10.5$, a comparison of Eqs. 1 and 2 shows that the circuit model predicts a sensitivity more than sufficient to achieve complete compensation at 77 Kelvins. That is, the tuning sensitivity of

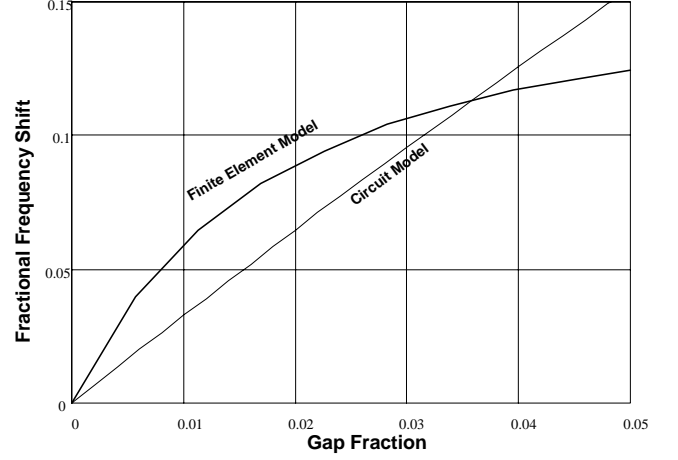


Figure 7: Gap Sensitivity Estimated by Circuit Analysis and Finite Element Calculation.

$\epsilon_s/\pi \approx 3$ is larger than the required value of $13.5/7$ from Eq. 1.

4.1.2 Gap Sensitivity Estimation by Finite Element Calculation

Figure 7 shows a comparison of the circuit model prediction with calculations using a recently developed finite-element methodology. This CYRES 2-D program[8] method takes advantage of sapphire's cylindrically symmetric dielectric properties to allow a simplified and more accurate calculation of whispering gallery mode frequencies and fields than was previously possible. The finite-element approach allows relatively complicated geometries such as ours to be easily treated.

As expected, this more accurate calculation gives a larger tuning effect than the (lower bound) circuit model prediction, but, somewhat surprisingly, shows an additional effect. As shown in Fig. 7, the finite element method predicts that tuning effectiveness (the *slope* of the curve) will be degraded for gaps as small as 0.02 of the resonator height.

This reducing sensitivity is shown to be due to a feature not included in our simple circuit model. For larger gaps (and also for large n , where the wide elliptical loops for E as shown in Fig. 6 tend to be narrow and tall instead) finite element solutions exhibit a substantial horizontal (azimuthal) electric field component near the gap, showing that the gap capacitance C_g is bypassed by azimuthal displacement currents in the

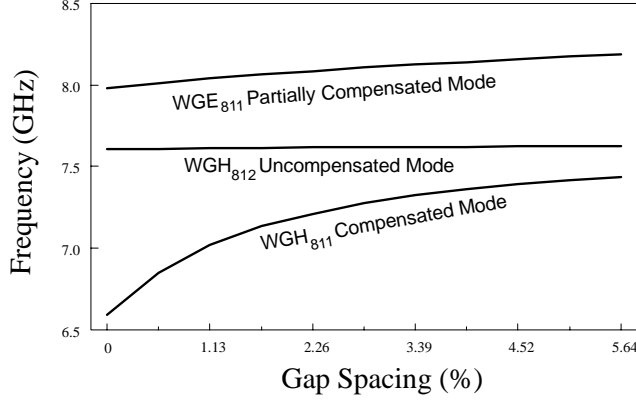


Figure 8: Frequency dependence on gap spacing for modes from various families (finite element calculation).

sapphire. That is, a more accurate circuit model would contain an additional capacitance C_{sg} acting in parallel with C_g .

The variation in tuning sensitivity with gap spacing provides a means to adjust its strength in order to match other requirements. Thus the gap may be varied to provide compensated operation in a particular mode at a particular temperature.

Not all mode families are found to be strongly affected by changes in the gap spacing. This is to be expected, for example, for modes with very small E fields in the gap region. Figure 8 shows the frequency dependence on gap spacing for three examples of modes with different characters for their electric fields. In order of decreasing sensitivity to gap spacing they are:

- The WGH_{811} mode with a maximum of vertical E field in the gap shows a rapid increase in frequency with increasing gap spacing;
- The WGE_{811} mode has a maximum of radial (horizontal) E field at the gap and shows a slight frequency increase;
- The WGH_{812} mode with a sign reversal of vertical E field at the center, and very small values in the gap shows almost no change in frequency.

These results quantitatively confirm that a strong vertical E field in the gap region, as exhibited by the WGH_{n11} mode family, and displayed in Figure 5, is essential to achieve high sensitivity to gap spacing.

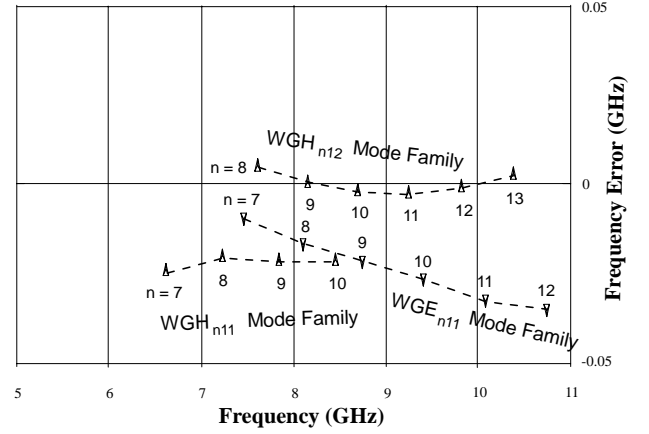


Figure 9: Frequency difference between experimental results and finite element calculations for several mode families.

For this geometry it is also the “fundamental” mode family, showing the lowest microwave frequency and highest mode confinement for any given azimuthal wave number n [1]. This family is thus an ideal candidate for use in a composite compensated resonator. Similar effects have been previously used to provide frequency variability for sapphire resonators [16].

4.2 Experimental Tests

4.2.1 Mode Frequencies

A resonator was constructed with configuration and dimensions as shown in Fig. 4, and with the parts mechanically and thermally bonded by means of pure indium solder. A clean (scraped) molten indium pool on each end of the copper post was mated in turn to an evaporated gold layer on the sapphire parts. After cooling to 77 kelvin, the frequency, Q , and coupling coefficient was measured for each of 69 resonant modes from 6.6 GHz to 10.75 GHz. This list was then preliminarily matched by frequency with the finite element data. Analysis of the electromagnetic visualization of the resonator cross section using the CYRES 2-D software conclusively identified the experimental modes for each family.

Figure 9 shows the excellent agreement between theory and experiment for the three mode families previously discussed. The data indicates a frequency difference of less than 0.4%.

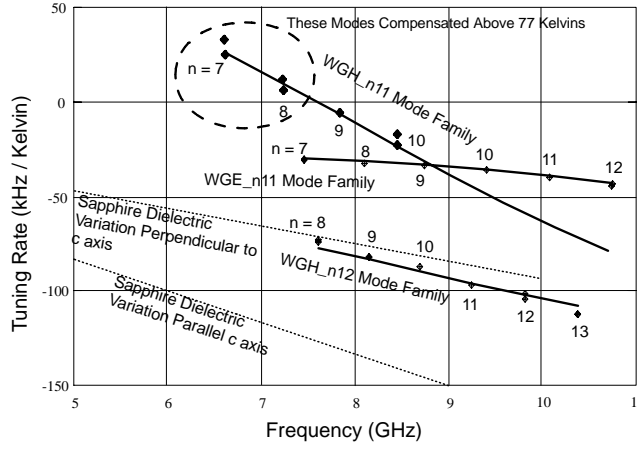


Figure 10: Calculated (lines) and experimental (points) temperature tuning rates at 77 Kelvin for the 3 dominant (lowest frequency) mode families in a sapphire/copper compensated resonator with a gap of .050 cm.

4.2.2 Temperature Tuning Rates

A direct demonstration of the effectiveness of our compensation mechanism can be shown by experimental measurement of the rate of frequency change with temperature at 77 Kelvins. As shown in Figure 10, the experimental points with positive values indicate modes that are actually overcompensated at 77 K. They will have turnover temperatures (complete compensation) above 77 K, as desired. Negative values indicates under-compensation or even no compensation (if values are approximately the same as expected from the sapphire dielectric variation alone), and a zero value would indicate a turnover temperature at exactly 77 K.

A comparison of calculated and measured tuning rates in Figure 10 shows excellent agreement. Sensitivity to small changes in the gap spacing was calculated with the finite element software. The results were combined with values for the expansion coefficients of copper and sapphire (Fig. 3) and a *fitted* value for the sapphire dielectric temperature dependence. As shown in Fig. 2, the dielectric variation values can be expected to vary between 9.4 PPM/Kelvin (perpendicular) and 16.75 PPM/Kelvin (parallel) at 77 Kelvins. However this represents data measured at kilohertz frequencies[12], and may be modified at microwave frequencies.

The fitted values were 11PPM/Kelvin and

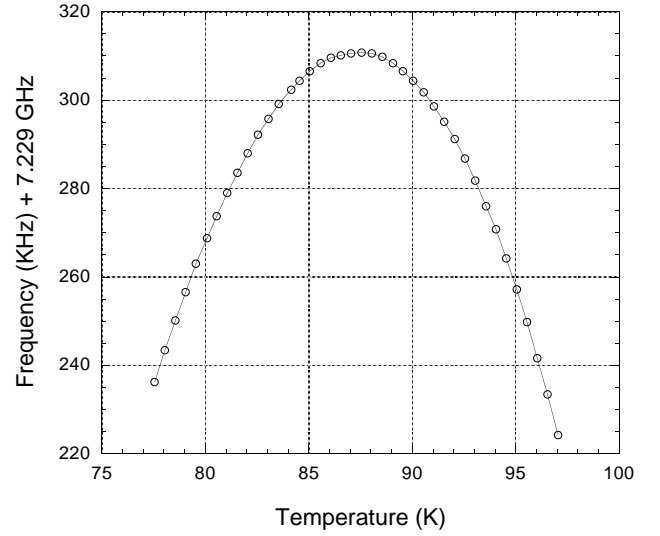


Figure 11: Temperature dependence of frequency for WGH_{811} compensated mode at 7.23 GHz.

10.5PPM/Kelvin for the WGH_{n11} and WGH_{n12} mode families, respectively, and 7 PPM/Kelvin for the WGE_{n11} family. It is to be expected that the WGE families would have a lower value, because the electric fields in this case are almost entirely in the $r - \phi$ plane, and so correspond to the lower “perpendicular” results, while the values for the WGH modes have a substantial fraction of their electric fields “parallel” to the z axis.

Both calculated and experimental values in Fig. 10 show a weakening of the compensation effect for higher mode numbers (higher frequencies) that was not predicted by the simple circuit model as discussed in relation to Fig. 7. Thus, modes with $n \geq 9$ in the WGH_{n11} mode family are under-compensated at 77 Kelvins, and so would require operation at a lower temperature. However the $n = 8$ modes at 7.23 GHz are just slightly over-compensated, and so can be operated at a temperature a little above 77 K, as desired. More detailed calculations show that higher frequency compensated operation is possible using a smaller gap spacing – where the compensation effectiveness is not so strongly dependent on the mode number.

4.2.3 Compensated Resonator Operation

One of the two WGH_{811} modes was chosen for further study. This mode showed the highest quality factor of any of the compensated modes with $Q = 1.8 \times 10^6$. Figure 11 shows a plot of the res-

onance frequency for this mode showing a turn-over temperature of 87.09 Kelvins. A quadratic approximation in the vicinity of the peak gives:

$$\frac{\delta f}{f} \approx 1.17 \times 10^{-7} (T - 87.09)^2.$$

A residual linear thermal coefficient due to imperfect temperature adjustment $\delta T = T - 87.09$ can be derived from the slope of the curve as

$$\frac{1}{f} \frac{\partial f}{\partial T} \approx 2.34 \times 10^{-7} \delta T. \quad (3)$$

Equation 3 allows us to estimate the thermal requirements that would allow such a resonator to achieve its ultimate stability of $\delta f/f \approx 10^{-14}$. If the temperature is held at the turnover temperature with an *accuracy* of $\delta T = 1$ millidegree, the slope given by Eq. 3 will be less than 2.34×10^{-10} per Kelvin, requiring a *stability* of 43 micro-Kelvins to achieve 10^{-14} stability. Accuracy and stability are distinguished in this discussion, because the δT accuracy needs to be held over a relatively long operational time period (possibly days or months), while the strength of the sapphire oscillators is in short-term stability. Thus, in order to achieve a stability of 10^{-14} for a time period of e.g. 100 seconds, the temperature would need to be stable to 43 micro-Kelvins for 100 seconds, but could vary up to 1 milliKelvin over the time period of operation. These requirements are easily met using conventional thermal regulation technology as developed for use by other types of frequency standards[17].

4.2.4 Oscillator Stability

An oscillator was constructed, stabilized by the WGH_{811} mode of the compensated resonator. Preliminary oscillator tests were accomplished with open loop control of the resonator temperature, and with the can temperature not regulated, but determined by direct contact with a liquid nitrogen bath. The stability of the oscillator was characterized using a hydrogen maser frequency standard as reference.

As shown in Figure 12, the Allan Deviation of frequency variation was measured to be $1.4 - 6 \times 10^{-12}$ for measuring times $1 \text{ second} \leq \tau \leq 100 \text{ seconds}$. There was a large but constant frequency drift during the course of the measurements which we attribute to the uncontrolled can temperature and the changing level of liquid nitrogen. It seems likely that the frequency variation observed was also caused

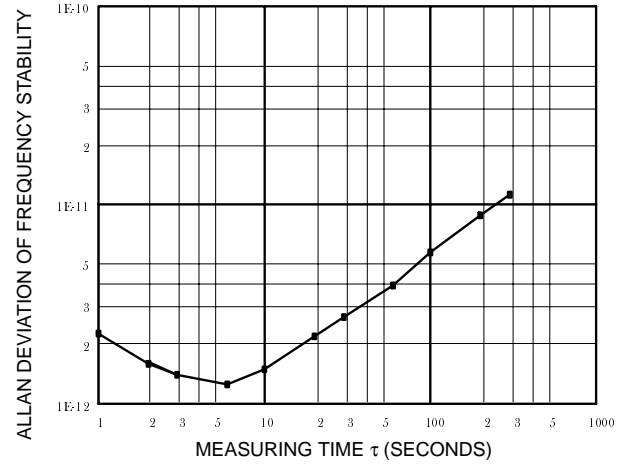


Figure 12: Frequency stability of sapphire oscillator with uncontrolled can temperature

by can temperature variations. We project a frequency stability 10^{-14} for this resonator with stabilized can temperature and with a mode quality factor of $Q = 10^7$.

5 Conclusion

We have demonstrated a new ultra-stable oscillator (USO) capability which promises performance improvements over the best available crystal quartz oscillators in a compact cryogenic package. First tests of this mechanically compensated sapphire oscillator show stability in the low 10^{-12} region for measuring times $\tau \leq 100$ seconds. We project a stability of 10^{-14} for this technology at liquid nitrogen temperatures.

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